RECONSTRUCTED MATHEMATICS IN THE HIGH SCHOOL.*

By Henry C. Morrison.

Few are satisfied with the present mathematics situation in the high school, particularly in the first two years of the high school. Dissatisfaction is found in the college faculties which deal with the product, among the mathematicians who are looking for a foundation for productive scholarship, among the teachers who are looking for something better; and dissatisfaction coupled with ridicule is found among the business men, engineers, and others, who expect mathematics learned in the school to function in the practical affairs of life. The purpose of this paper is an attempt to analyze the situation, to find out what is the matter with mathematics in the high school, and if possible to throw some light on the way out.

Three Factors Involved: Students, Social Needs, Available Subject-Matter.—Whatever the solution ultimately may be found to be, it can confidently be stated that the three chief factors of the problem to be solved here, as in the case of all other curriculum problems, are: (1) the pupil and the laws of his mental growth and development; (2) the social needs which the school as an institution must serve; and (3) the availability and use of the material under discussion—mathematics in this case—as an instrument for such pupil development and his adjustment to such and such social needs or purposes.

The existing mathematics of the high school, and particularly that of the first two years, however taught, falls far short of satisfying the known laws of adolescent growth, and it bears little relation to any known social needs. Referring to existing mathematics, the writer of course has in mind first of all the formal algebra and geometry usually found in the first and second high-school years; and to these courses may be added

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the solid geometry, trigonometry, and advanced algebra commonly taught in the last two years.

Subject-Matter Must Function Throughout the Process of Learning.—You may teach the pupil much or little, but what you really teach will depend entirely upon what he can and will learn. It is simple enough to cram a youth with learning which will enable him creditably to pass off a recitation or an entrance examination. That depends upon the force and skill of the teacher. But to ground the pupil in learning which will react to the only real test, namely, "will it function?" depends as much upon the nature of the pupil's mind and the stage of his development as upon the professional tact and skill of the instructor.

There is nothing to which most processes in algebra or geometry, or indeed arithmetic, can be applied except to more algebra or arithmetic. Hence, while the pupil may for the time being attain perfect marks, his learning becomes no part of his stock of usable ideas, and he straightway forgets all about it until he is put through a naïve "review," which in its turn needs to be "reviewed" when he becomes a college Freshman or enters a shop. The first conclusion then is that we must find a kind of mathematics material not only which will function but which does function in some other field than mathematics while it is being taught, and such use must further respond to a real need felt as such at the time by the pupil. So only can mathematical concepts become realized.

Disciplinary Arguments Not a Sufficient Justification.—Unless we can find some other justification for courses in the high school, many of them will undoubtedly presently travel the road of Greek, and we shall have little that can be called education left.

It is fairly to be assumed at this day of the world that unless a course can justify itself as offering to the pupil a system of ideas which help to interpret to him his environment and enable him to react to new and strange situations in that environment, then such a course has little place in a modern educational institution.

But laying aside the purely disciplinary argument in its extreme form it may be objected, with reference to geometry

especially, that here is a method of thought in which the educated man should be trained. The contention might be granted in part if the thinking of the modern world were done in the form of the syllogism and in mathematical terms as was once the case. The fact is that the thinking of the modern world is done mainly in inductive form and in terms derived from biology. Let us consider the pupil and find out what we know about him.

In the first place, the youth when he comes to the high school is, and has been for about two years on the average, an adolescent. If a boy, he is a clumsy, awkward chap, who has lost all the nimbleness and agility which he had three or four years ago, and is now chiefly occupied physically in keeping from "falling over himself" and in keeping out of sight. Mentally his mind is dreaming and seeing things never dreamed of before. If a girl, well a mere man had perhaps best not try to do justice to her. Probably, in her own way she is at bottom in the same state as the boy, though she can laugh, or at least giggle, it off, while he cannot.

It is the worst period for anything like drill. It is a period when new ideas, especially those of a general spiritual type, are entering the opening mind in hosts of new forms; when the physical organism is undergoing a process of complete reorganization and readjustment; and when mental attitudes and powers are undergoing a similar and corresponding change.

Now, it does not at all follow that the adolescent boy or girl is incapable of mathematical concepts, or necessarily finds them distasteful. A prominent characteristic of the mental attitude of the adolescent is an openness to entirely new types of ideas as well as an entirely new set of reactions. It is very likely true that there is in the adolescent mind a capacity for apprehending new mathematical concepts of a much higher order than has generally been thought possible. In any case, it does seem to be true that the ability of the mind to apprehend new ideas is related to the ideas already actually in mind, and the ability to assimilate new notions and make them a part of the intellectual capital is largely a question of opportunities for such ideas to function in the interpretation of some feature of the environment.

The difficulty with the present high-school mathematics, especially algebra, is not in the intrinsically abstruse character of the concepts, but rather (1) in the extreme difficulty of finding an opportunity for them to function, and (2) in the fact that the algebra as taught is almost entirely an organizing and drill subject.

It isn't that the pupil is not capable of a high order of thinking; he simply hasn't had the experience with which to do his thinking. He is eager and anxious for new ideas; he never will be more so; but he cannot effectively formulate ideas which form no assimilated part of his intellectual equipment. The same boy will perform marvels of wireless telegraphy the understanding of which he has gathered from his juvenile periodical, but he will gaze stupidly at his science teacher who talks to him of the elementary units of electricity, and ultimately fail in his examination.

Fundamental Attitudes of Girls Are Even Less Favorable for Abstract Mathematics.—In our analysis of the difficulty which seems to exist in the present mathematics situation, there are important special features true in the case of girls which are not true in the case of boys. Whatever is true of the mental attitude of adolescents in general to mathematical culture, it is also true that boys are normally organized to react favorably to the functions of which mathematics must become one of the chief instruments of interpretation. The woman on the other hand is organized both bodily and mentally for dealing with an entirely different set of functions, in which mathematics plays a small part. At this particular period she must be full of new ideas and insights totally different from those which are coming to the boy of the same age.

Again the high school is conspicuously the institution in the whole course of education which is today in an unsettled state. The ancient landmarks have been torn up and the boundaries are in process of revision.

Conditions have changed. A generation ago the high school was an institution which few pupils reached. Life was relatively simple and the common school education was felt to suffice for the great majority. But since the eighties of the last century high-school enrolment has been outrunning population

in growth all over the United States. This fact points unerringly to the conclusion that the expression "common school" must be extended and applied to the secondary school. The high school is rapidly becoming on the whole not the "people's college," but a part of the educational scheme common to all.

These changes have altered greatly both the curriculum problem and the pedagogical problem in the high school.

Differentiation Now Should Fall at Beginning of Adolescenses; Not at End of Compulsory Period.—The still prevalent eighth-ninth grade division point is probably related to a process of evolution which had gradually brought about a completion of eight years of work at the average age of fourteen, when by common agreement in most states the age of compulsory education has ended. Twenty years ago the Committee of Ten foreshadowed what is rapidly coming to be seen to be a fundamental principle, namely, that the division point should come at the dawn of adolescence rather than at its most critical point. This corresponds very well to a division point between the sixth and seventh grades, which teachers have for a long time suspected to be the right one.

In Any Course, Detailed, Concrete Aims, Related to Social Needs, Must Replace Formal Aims.—The plain fact is that every school in every age has been at bottom an attempt to adjust its pupils to the requirements of the society in which they live. The state as the will of society bids us get back to the original purpose of the school, namely, getting pupils ready to live effectively in our own twentieth century United States—not in the eighteenth century, nor in Germany.

Cultural Course Related to Contemporary Needs Will Continue to be Prominent.—Your curriculum may still be strictly educative or developmental, or it may be technical with a view to immediate special training for life-work or vocation.

Mathematical Courses Should be Differentiated for Cultural and Technical Purposes.—The solution on the side of the technical high school should in principle be very simple, to wit: the thorough teaching of such processes as are needed in the industry for which training is given followed by drill to the point of efficient functioning within a narrow range.

But because, as it seems to me the facts indicate, the cultural

high school is now, and will increasingly continue to be, the type of secondary school which the American school man will have to administer and in which the majority of our secondary teachers will work, I shall deal with that type only.

Development of Adaptability in Adolescents is the Aim.—Adaptability is the standard by which all mental development above the level of tropism is to be measured.

To be more concrete, I mean that the modern American high school must produce a young man or young woman, not necessarily with specific training, but capable of intelligent adaptation in any situation in which he or she is likely to be placed.

Again, lest we forget, let it be observed that the developed capacity of the individual to react to a strange situation is a question of his possessing a working system of ideas, and not of his having exercised interminably a mythical mental faculty.

Some Elements of Knowledge Are Common to All Zones of Adaptability.—For instance, knowledge of the human body, the heritage of the race in various institutions and a racial literature, in art, in ethics, and so on. Some elements are common to two or more zones of adaptability, as, for instance, the biological sciences to the housekeeper and the agriculturist. But the specific elements which go into an understanding of the fundamental problems of the homemaker are widely different from those needed in the educational equipment of the engineer or the attorney.

One of the chief functions of the secondary school is and must necessarily be the furnishing of opportunity for a selective process to take place upon the native bent of a pupil, to discover to each so far as possible the broad zone within which his future activity will normally lie.

High-school Curricula Should be Differentiated to Parallel Broad Zones of Adult Activity.—In the larger cities this division has already been foreshadowed by the erection of distinct types of high schools, to wit: the classical high school, the high school of commerce, the mechanic arts high school, and latterly the domestic arts high school. Similarly there has recently been developed in rural communities the agricultural high school.

Then there is a picture of the well-developed program of

today in a high school, enrolling say 250 pupils, and located in the typical community of say 10,000 people with industrial interests ranging all the way from a zone of farms a few miles out to several highly developed industries in town. Such a school should offer well-differentiated curricula calculated to furnish the educational foundations for: (*a*) homemaking and housekeeping; (*b*) agriculture; (*c*) mechanical and engineering pursuits; (*d*) commerce; and (*e*) the professions through its college preparatory curriculum probably reorganized somewhat in both content and method.

There Should Be Appropriate Mathematics for Each of These Curricula.—For the girl engaged in acquiring the educational foundation for her normal life-work, but little mathematics beyond the simple arithmetical computations which she has brought with her from the elementary schools will be needed.

The mathematics of the educated farmer is, first, a good deal of practical arithmetic, but not involving any very abstruse processes; second, a good conception of the properties of plane and solid figures; third, plane trigonometry and surveying. All of these the student will use in his studies and of them he will make frequent use in his vocation.

In commerce, arithmetic and certain of the processes of algebra applied to the solution of practical commercial problems will be needed.

In the mechanic arts there is an extremely interesting field for much more mathematics than we now commonly find in the secondary school. Arithmetic enough the boy already has. He needs algebra enough to understand more useful processes, and will use constantly a considerable range of constructional geometry. But more than that, his work will give him a concrete basis for trigonometry and the elements of calculus, the latter a perfectly feasible high-school subject when taught in connection with other studies and with shop work in which it has a constant opportunity to function, as I shall attempt to show later.

The mathematics of the college-preparatory curriculum will of course relate itself to mathematics in the college, until colleges conclude to relate their mathematics to what can be done in the preparatory school.

Special Provision May be Made for Brilliant Students of Mathematics.—Probably, from 1 to 1½ per cent. of all high-school students, take them as they come, have some incipient talent of this type. All large high schools, say those enrolling 500 or more, should, I believe, provide special courses for divisions of these people permitting them in every way to fulfil their bent. In other smaller schools, it is a pretty poor teacher who will not gladly put in extra time with these brilliant minds.

There remain two other considerations related to the social purpose of the school as an institution which must be considered.

Moral Purposes.—The first of these is the moral purpose of the school. Mathematics particularly has been thought to have a special moral or quasi-moral purpose in the school on account of its excellent adaptation to disciplinary ends.

But Mathematics as Such Can Contribute Little to Moral Training.—For essential morality is I think a question of the relations of individuals in society, and all we mean by moral education is the adjustment of the pupil to the standards of life in society sanctioned by the highest social ideals of his time. And this is not a matter of book learning, but rather arises, if it arises at all, from the interaction of the various personalities composing the school, especially of course from the reactions of the pupil to the personality of his teachers.

There is one more precious feature of the mathematics of the secondary school, namely, the use of mathematics for "molding the mind of the pupil in exact methods of thinking" which is the citadel of the disciplinary position. It has been, I think, amply demonstrated that the mind which has been molded to the method of mathematics will use that method in mathematics, and in thinking allied to mathematics, alone. The mathematician himself behaves in about the same manner as other mortals in a social or a political situation, but he reacts more efficiently in a certain type of scientific situation than does he who is devoid of mathematical training. The "method of mathematics" is a highly desirable asset to certain types of education, but the method will certainly not be acquired through a period of abstract study of algebra and geometry. It can only be acquired through the constant functioning of the mathematical processes learned, in the interpretation and solution of problems presented by other subjects.

Critical Evaluation of Material Available in Mathematics.

It has already been stated that in the analysis of any program problem presented by the school, three factors must be considered. First, the nature of the pupil must be known. Second, the general purpose of the school as a social institution must be investigated. Third, the availability of the material under discussion must be criticized.

What has mathematics to offer which is essential and valuable in the process of enabling pupils to interpret new and strange situations in which they will be placed.

The Use of Mathematics as a Tool in Scientific Thinking is Most Important.—First of all, mathematics like language is in the main a "tool subject," and not one possessing inherent value of its own. There are perhaps few better criteria of the trained mind than its distinguished ability to use mathematics as an instrument for the mastery of scientific truth. The educated man endeavors to reduce all his important experiences with the material world to mathematical terms and thus to proceed confidently from step to step in his career. The uneducated man never knows exactly what his experience means, and proceeds by guess in the administration of his affairs, with great waste of energy and of substance and with a high percentage of failure.

Differentiated Courses are Needed to Provide Opportunities to Use Mathematics as a Tool.—In the first place, the traditional round of algebra, geometrical logic, advanced algebra, and trigonometry ought to be entirely abandoned and a fresh start made. Entirely different sets of mathematics material should be organized for domestic arts, for agriculture, for mechanic arts, for commerce, and for other new curricula or schools as they may be organized. In each of these several schools, mathematical processes should be taught only as fast as they are needed, but the need should be sought out and brought forward as well for the sake of the intellectual value of the subject under instruction as for the sake of the pedagogy of mathematics.

To take up each curriculum in turn.

In domestic arts, mathematics is needed to a greater or less

extent in dressmaking, in the study of house construction and of the apparatus of the household, in household accounts and other economic courses, and in the study of food values. But the mathematics needed nowhere reaches beyond the elementary principles of arithmetic and a moderate amount of mechanical drawing.

Geometry, Trigonometry, and Some Algebra in Agricultural Courses.—In the agricultural courses, for the measurement of their fields, in the laying-out of highways and drains, for analyzing the strength of the different members of buildings, for determining the profit of different fields and domestic animals, the pupils will need mathematics as a "tool" to enable them to read the situations presented to their intelligences. Then, what mathematics? Chiefly geometry and trigonometry and the art of making simple mathematical records and analyses; and out of these grows the need of some algebra.

The geometry which the educated farmer needs is the "earth measurer," not a system of logic. He needs an understanding and knowledge of the properties of plane and perhaps solid figures learned in exactly the same manner in which he learns the properties of soil in his soil physics. The geometry which is a study by constructive process with pencil and compass, with square and dividers, of the essential underlying principles of the science, is the geometry which will function and read out to the pupil the truths of which he feels the need.

The algebra needed, that is, the algebra which will function in this curriculum, centers around the equation.

Master Use of the Equation and Subordinate Processes.—Now to acquire facility in the use of the equation means a very considerable amount of practice, but such practice should of course consist in throwing into the form of equations statements which it is desirable to have in such form, and not in the solution of puzzles in the form of "problems" totally unrelated to experience. The competent use of the equation of course implies facility in the use of a limited number of other algebraic processes, to wit: the elementary concepts of algebra, the four fundamental processes with the shorter forms of multiplication and division, the simpler cases of factoring, the extraction of the square root (but not the cube), and probably an acquaintance with the essential principles of expressions in radical form.

At What Age Should Algebra and Geometry be Begun?—There is probably little or nothing in the way of introducing the type of geometrical study which I have described at any time after about the twelfth year, but the earlier the better. In the case of algebra, the unsettled state of the pupil beginning with about the age of twelve, culminating at perhaps the age of fifteen and fading into relatively settled conditions from that time on, makes attempts to develop facility in execution very unpromising before, let us say, about the age of sixteen.

Trigonometry enters at an entirely suitable period as now at about the age of seventeen or say in the eleventh or twelfth grade.

The Course Should Provide Constant Practice in the Mathematics of Records.—In the agricultural curriculum, and to a much greater extent in the commerce curriculum, is the opportunity and need of what may perhaps be called the mathematics of records. As pointed out before a characteristic difference between the educated and the uneducated man is the extent to which the former reduces his experiences to mathematical form, and reads their meaning in mathematical terms. The pupil should certainly be familiarized from the beginning of the secondary period with the practice of graphic expression and the reading of graphs.

Commercial Curriculum Presents Special Problems and Opportunities in Securing Educative Content.—A curriculum in commerce definitely and seriously organized for any purpose more worthy than as a temporary abiding-place for pupils of small ability will certainly offer broad scope for much mathematics—for more of the higher mathematics as now taught probably than any of the other curricula. This becomes at once evident when we contemplate what is involved in the rational interpretation of statistics, in the records of complicated transactions, in the understanding of banking, currency, and kindred questions which must of necessity be matters of daily experience to every really educated business man—not merely the occasional financier, but every small trader as well who would conduct his business intelligently.

The mathematics demanded by the situation and teachable in the secondary school appear to the writer to be at least the

following: (1) the science of accounts; (2) the principles of statistics; (3) the properties of number as set forth in the higher arithmetic and algebra.

Science of Accounts.—Bookkeeping, in the hands of a competent teacher, even now satisfies more of the requirements of the educative process than most high-school courses.

Principles of Statistics is Necessary.—The business man must read a very considerable part of his literature in statistical form.

To mention subjects like insurance, returns on investments, annuities, and similar considerations is to justify the need of the third kind of mathematics mentioned above, namely, the higher arithmetic and algebra or the study of the properties of number as such.

One final study is perhaps here worth while, to wit: a brief survey of the mathematics indicated for courses in the mechanic arts, for this curriculum certainly offers the broadest scope for mathematics teaching, though in the mind of the present writer commerce promises distinctly the greater intensiveness.

The mechanic arts curriculum ordinarily embraces: woodworking of a somewhat advanced type; forging; pattern-making; moulding and casting; and general machine-shop practice with the engine lathe, drill press, bed planer, and milling machine. To these must be added mechanical drawing of a more advanced type than that found elsewhere.

Now the underlying mathematics which will interpret the subject-matter of this curriculum and which will function pedagogically during the teaching process and which is probably assimilable during the secondary period is the following: (1) geometry, plane, solid, and descriptive; (2) the elements of plane trigonometry; (3) the elements of analytic geometry and calculus. Of course as mathematical tools there must also be the modicum of algebra which is really needed for reading purposes; and acquaintance with the manipulation of such devices as the slide rule and logarithmic and other tables, but not necessarily any great facility in the use of these appliances.

To a very considerable extent the educational level of the mechanic arts course will probably depend upon the degree to which the school succeeds in bringing mathematics to bear upon

the study of the various operations involved, especially in the machine shop in the later courses of the curriculum.

When the problem of the reconstruction of our mathematics is approached from the standpoint of analysis of the situation in a scientific spirit, much as has been the case with the scientific management people in the field of industry, a wonderful saving of time and a most fortunate enhancing of teaching efficiency is very likely to be the result.

SUMMARY.

The conclusions of this paper may be summarized in the following terms.

1. The traditional round of mathematics in the high school, to wit: elementary algebra, plane and solid geometry, trigonometry, and advanced algebra, must be revised both as to organization and content, and adapted to the known nature of the adolescent and to the social purpose of the high school as that purpose is increasingly revealed by modern conditions.

2. Mathematics must be treated primarily as a language, the purpose of which is the interpretation of the various sciences.

3. Courses in mathematics must be arranged at such points in the curriculum as will give immediate opportunity for functioning.

4. The several integral parts of the program such as the household arts, etc., must each have its own specially organized mathematics; and the mathematics of each curriculum should probably be in charge of the specialists of that curriculum rather than in the hands of a separate mathematics faculty.

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